



# Behaviorly realistic simulations of stock market traders with a soul

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## Abstract

The price fluctuations of the stocks in the financial markets are the result of the individual operations by many individual investors. However for many decades the financial theory did not use directly this “microscopic representation” of the markets. The main difficulties preventing this approach were solved recently with the advent of modern computer technology: – massive detailed data on the individual market operations became available; – “microscopic simulations” of the stock markets in terms of their individual participating agents allow very realistic treatment of the problem. By taking advantage of the modern computer processing and simulation techniques, we are now able to confront real market data with the results of simulating “microscopic” realistic models of the markets. These models have the potential to include and study the effects on the market of any desired feature in the investors behavior: departures from rationality, herding effects, heterogeneous investor-specific trading strategies. We propose to use the comparison of computer simulations of microscopic models with the actual market data in order to validate and enhance the knowledge on the financial behavior of individuals. Moreover we hope to explain, understand (and may be predict and control) macroscopic market dynamical features (e.g., cycles of booms and crashes, investors wealth distribution, market returns probability distribution etc.) based on realistic models using this knowledge. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Microscopic simulation of financial markets

Traders operating in the stock market behave in various manners depending on their individual preferences, expectations, wealth, memory and data processing capabilities. Yet, in order to obtain analytical results, many macroeconomic models were forced to assume a “representative” agent. While for very long time spans and very coarse averages such an approach (similar to the “mean field” in Physics) might have some relevance, it is sure to become unpractical for the detailed study of short times market fluctuations.

Microscopic simulation (MS) as an alternative to the representative individual framework is suggested

by similar techniques applied in Physics where, e.g., the global formulae for the gas behavior (of the type of the Boyle–Mariotte law  $PV = KT$ ) were substituted by the detailed simulation of the motion of each of the gas molecules (including their individual collisions and interactions and their interactions to the walls of the container and to the environment).

MS is used presently in many fields outside physics (ecology, immunology, cognition, marketing, psychophysics, etc. [1] in order to study the macroscopic behavior of complex system composed of many basic interacting elements. In MS, the microscopic elements and their individual actions are simulated by computer according to microscopic rules. The simulation can be thought of as an acting out of reality by the

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computer. In contrast to the purely analytic methods MS allows the study of markets with a macroscopic number of agents with inhomogeneous behavior which is not describable in terms of analytic formulae. Any financial system with arbitrarily realistic (or unrealistic) agent behavior can be simulated.

We believe that MS can be as fruitful in finance and in economics as it has been in physics, biology and sociology. The present proposal is focused on MS which mimics very closely the human financial behavior and the complex market conditions. More precisely we propose to simulate microeconomic models of the stock markets which include the known behavioral finance facts and to compare the results in detail with the high density (to the level of single trade) stock market data. In particular, we will analyze the effects of various behavioral rules on the macroscopic market behavior and on the individual welfare. We expect that some of the results of the Modern Portfolio Theory will survive (at least) in certain limit cases while some others might be seriously modified. It would be interesting to find out to which degree the market portfolio, optimal market hypothesis and the risk premia formulae are preserved.

## 2. The LLS model

We now first present a model [2] (LLS from now on) which was already studied and has proven quite adequate and then we will put it in a wider perspective and indicate the extensions which we propose in order to achieve a precise and detailed dialogue with the actual market data.

The microscopic “elements” in LLS are the individual investor. Individual investors interact via the buying and selling of stocks and bonds. The bonds are assumed to be a riskless asset (though default instances can be simulated) while the stocks are risky assets. In the original LLS, the investors are allowed to revise their portfolio synchronously at given time points, however, we performed already experiments with an extension in which the various investors may act sequentially at different times.

The bond yields a constant return  $r$  at the end of each time period. Thus if investor  $i$  invests at time  $t$  his/her entire wealth  $W_i(t)$  in bonds then at time  $(t + 1)$  will be  $W_i(t)(1 + r)$ .

The return on the stock is composed of:

- Capital gain (loss): The price  $P(t)$  of the stock varies in time (driven by the supply and demand collective dynamics described below). Consequently, if the trader  $i$  holds  $N_i(t)$  shares at time  $t$ , his/her loss/gain at time  $(t + 1)$  will be

$$N_i(t)(P(t + 1) - P(t)).$$

- Dividends: The company earns income (a random variable) and distributes dividends  $D(t)$  per share at time  $t$ .

Let us now follow the dynamics of the market by looking at the market evolution as feedback process relating the individual decisions/operations/gains/losses to the global market price variations: Before the trade at time  $t + 1$ , the total wealth of investor  $i$  is (due to the dividend and bond yield):

$$W_i(t) + N_i(t)D(t) + (W_i(t) - N_i(t)P(t))r.$$

The form of the market order submitted by each LLS trader before time  $(t + 1)$  is in the form a function  $N_i(PH)$  representing the amount

$$N_i(PH) - N_i(t)$$

of shares (s)he wants buy/sell as a function of hypothetical price  $PH$  at time  $(t + 1)$ . As the number of shares in the market, denoted by  $N$ , is assumed to be fixed, this will determine the equilibrium price  $P(t + 1) = P$  by the equation

$$N_1(P) + N_2(P) + \dots + N_I(P) = N,$$

where  $I$  is the number of investors in the market.

In order to decide on the optimal  $N_i(PH)$  each investor has to find the optimal diversification between the risky and the riskless asset. In LLS, the investor “well being” was expressed by an “utility” which was assumed to be a given (investor specific, increasing, concave) function  $U(W)$  of the investor’s wealth, reflecting his/her personal preference. The objective of each investor was to maximize the expected future value of his/her utility. In order to decide the optimal function  $N_i(PH)$ , each trader  $i$  takes into account that immediately after the  $(t + 1)$  trade there is capital gain/loss only on the  $N_i(t)$  shares (s)he held before the trade  $(t + 1)$  and not on shares bought or sold at the time  $t + 1$  trade. The investor decision  $N_i(PH)$  before trade  $t + 1$  affects how his/her eventual (hypothetical) wealth  $WH_i$  is going to be divided at time  $(t + 1)$  in

order to maximize his/her expected utility  $U(W(t+2))$  at the next period, time  $t+2$ . Since the future returns were not known in advance, (s)he has to “guess” them: (s)he uses the ex-post prices

$$P(t), P(t-1), \dots, P(t-K_i)$$

as an estimate for the probability distribution for the ex-ante price  $P(t+2)$  (we call appropriately  $K_i$  the “memory length” of the trader  $i$ ).

One of the major changes we propose for the future research is to use the Value function of the prospect theory and also to allow for other factors to affect the “subjective” investor “well being”: to be proven “right” in one’s past decisions, intuitions, to go (“heroically”) against the trend, to be “loyal” to a “solid” company, not to perceive oneself as a “sucker”, etc.

The LLS model sketched above and its implications were studied recently by a few groups [3]. The extensive computer experiments lead to a wide range of realistic features and illuminating insights. On the other hand, by comparing with the actual data, one was able to identify additional crucial elements which had to be integrated in the model in order to make it conform reality.

It is hoped that this is a solid basis for modeling in increasing degree of detail and precision the realistic market dynamics. However, the simple LLS model is not expected to give detailed predictions and explanations of the “microscopic” single-trade-frequency market dynamics.

### 3. Extending LLS to behavioral finance

In order to introduce in microscopic simulations behavioral finance information one has to start from LLS or from other recent microscopic models (Lotka-Volterra [4–6]) and perform the necessary changes/extensions.

In order to explain how the LLS model can be extended to include the behavioral finance features, let us reformulate LLS along the flow chart in the Fig. 1.

We explain for each module in Fig. 1 first how (if at all) it is represented in LLS and then, how it can be modified for more realistic modeling.

**History.** In LLS the history consisted just of the past Prices. In more sophisticated extensions one can

include: internal and external events, tentative posted bids (rather than actually effectuated transactions), records of the identity of the trader making a certain transactions (some traders (“Soros”) can influence the bidding of other traders).

**Memory filter.** In LLS the only filtering is a memory length cut-off: each of the investors is characterized by a memory length. They recall data during the last  $K_i$  trades. In other models one can recall past moving averages or recalling better and for a longer time events in which (s)he was personally involved (in an open position) which resulted in dramatic losses or events coincidental to other dramatic events.

**History as subjectively recorded.** The result of the filtering may depend on present wealth (large gains worth memorizing for an average investor may be considered worth forgetting by a rich one). Also one tends to forget defeats (unless they result in very painful losses). In LLS, the recorded memory is an undistorted list of the past  $K_i$  gains.

**Procedure for estimating the probability of various future events.** LLS associates to future events probabilities equal to their relative occurrence in the recorded past (i.e., within the memory span). In another version, LLS associates progressively lower probability to old events. In a behaviorally oriented simulation, one can introduce the following features:

- associate high probability to dramatic events which occurred in the past (especially if they affected individual),
- associate 0 probability to low probability events (or in general clustering values: the famous 0.99 effect).

One can also use chartist methods to guess the probability for the price to go up or down. One can also introduce irrational beliefs, dependence on (factually related or unrelated) external events evolutionary selected estimation procedures, etc. Another illusion affecting the probability estimation is the “control-over-the-market” which leads to low weight for statistical evidence and high weight for personal observations.

**Subjective probability associated to the various future events.** The result of the probability estimation procedure may take the form of probability for respectively up or down price moves. In LLS the result is

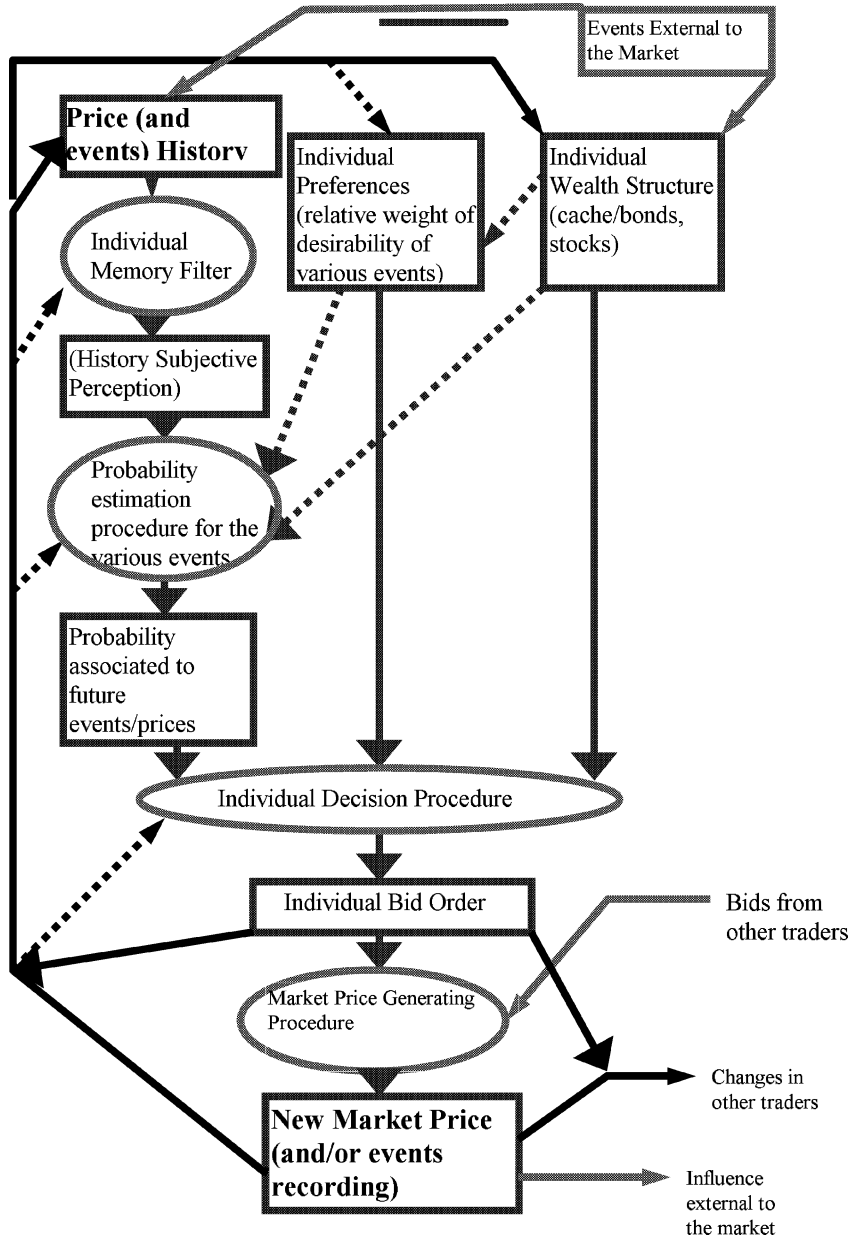


Fig. 1. Generic flow chart for a microscopic simulation of financial markets.

a list of typical events and their associated expected probabilities. In other case one may give detailed case by case probability lists for complex event sequences. A typical way to express expectations is to generate a probability distribution curve (for future market price).

**Individual preferences.** Each trader associates to each event a measure of (un-)desirability. The format for expressing it may be like in LLS an utility function relating a desirability measure to each wealth level (one then tries to optimize this expected “desirabil-

ity”). Another method, quite similar is to use a “Value function” which depends also on a “reference point” (e.g., the present wealth) and possibly on the wealth history. In more realistic simulations one has to include case by case preference labeling which would depend in more detail on previous personal operations and market gains:

- the famous Kahaneman–Tversky case of “(not) repurchasing lost movie-tickets”, or
- the desire to be “proven right” (and incurring cognitive dissonances, guilty feelings and overreaction), or
- mental compartmentation of wealth (e.g., considering as reference wealth value the presently risked amount due the present position, or
- the total present amount in shares rather than the total personal wealth).

An important “reference point” for the price fluctuations might be the market price at which a position was opened. Another “reference point” might be the index at the beginning of the day or the traders’ average revenues. The mental compartmentation appears also in clustering of stocks (according to their beta’s, the company type of the trader’s affinity to the company’s product). This may lead to the similar treatment of all the stocks in such a cluster.

**Individual wealth structure.** In LLS the wealth consists in just bonds and shares and changes only through interest, dividends and share price changes. In practice it might be relevant for the trader decision how much wealth is in easily liquidable assets. (The tendency to liquidize is often associated to “crisis news” and might lead to severe fluctuations similar to phase transitions in statistical physics.) Moreover the wealth may be affected by external factors: taxes, overhead, inheritance, current expenses, coupling to events.

**External events influence.** In LLS one considered only the effects of unexpected changes in the timing (delays or advances) in the dividend payments (and amplitude and duration of their effects were measured). In addition one may introduce also knowledge on mergers, take-overs, national bank interventions, inside info, “comet” events, etc.

**Individual decision making procedure.** In LLS each agent averages over the Utility of the various future

events with weights proportional to their subjective probability. Then one maximizes this function with respect to the amount to be presently sold or bought by the individual. In LLS we allowed for some amount of irrationality by introducing random variations from the optimal value obtained above.

Similar procedures can be applied to a Prospect theory-like behavior by computing the Value function for each event, averaging appropriately and maximizing with respect to the present individual market operation.

Irrational decision procedures such as throwing a coin or applying random chartist rules (may be evolutionary selected) can be applied too. Also the trader may relate bid decisions to external events apparently unrelated to the market (“comet news”).

One can also introduce fundamentalist decision criteria related to “real value” or criteria inspired by CAPM by giving a premium for low past volatility (though this may be already implicit in the choice of a concave Utility function).

The very treatment and use of different decision procedures should be considered. The trader might either be committed to one procedure or vacillate between a few alternatives or combine them in its behaviour.

One possibility is to let external events become triggers for the various procedures. Moreover different procedures might be used for different time scales: somebody may be a fundamentalist on very long range, chartist at daily scales and completely random in intra-day gambles.

The mental time division into fixed periods may induce various behaviour related to the beginning and the end of such a period. In particular, somebody who incurred losses at the beginning of such a mental time division (fiscal year, week, day) may become risk averter for the entire rest of the period (and the opposite: the “House Money Effect”).

Another aspect of the investing strategy which became wide-spread among fund managers and traders is to follow procedures which can be defended in court in case it leads to disastrous losses by the client(s).

**Individual bid order.** The result of the decision procedure can take the form of a firm order or, as in LLS, a graph relating the quantity to be bought/sold to the price at the transaction time. Another way,

fit mainly for agents modeling banks or other large financial institutions is to post limits: buy below X, sell above Y.

**Market settling mechanism and operations.** The market rules have a major importance for the liquidity, depth and breadth of the market.

In LLS, the bids (in the form of graphs connecting the desired/offered volume to the market price at the trading time) are collected and the price is fixed by the market clearance condition (the price at which offered and demanded quantities are equal).

One can alternatively consider binary trades between matching buy/sell traders pairs (which find one another following posted offers).

A simpler effective way to relate demand/offer to the change of the price is to assume that each sell/buy individual operation lowers/rises the price by a quantity proportional to the volume of the operation.

**Feedback influence of market posted information.**

In LLS the memory filter, the preferences, the probability estimation procedure and the decision procedure are fixed (though different) for each trader. In principle one can allow those procedures to change in time (e.g., as a result of “learning”).

Some of the mental mechanisms which may lead to such changes can be carried over from the known behavioral data, some others may be assumed to undergo an evolution under “natural selection”. Yet, for some of them one will have to make assumptions which will be validated or invalidated by the detailed comparison to the experimental data.

This very detailed intensive and prolonged dialogue between microscopic representation modeling and tick-by-tick fully documented operations is the heart of the present proposal.

#### 4. Further computer experiments and market measurements

The measurements we performed until now were limited to standard statistical measures of the probability distributions of individual wealth, market returns, volatility time correlations and transactions volumes. In order to gain detailed knowledge on the microscopic dynamics driving the market behavior, one has to de-

sign computer experiments and measurements closer to the financial meaning of the data.

For instance we attach significance to measurements of the time duration that investors put themselves at risk by keeping a position open. Measurements uncovering the tendency of large traders to divide large transactions in smaller portions in order not to influence adversely the market price by their bid.

The dynamics of volatility clustering and trade volume clustering and their causal relation will be also one of our focuses. Moreover we will try to understand the behavioral mechanisms which underlie the persistence of trends and their reversal. Simple time autocorrelation measurements might not be enough and more sophisticated methods design to uncover hidden market behavior patterns might be necessary. This brings to the central issue of market efficiency: while macroscopically it is obvious that major arbitrage opportunity are taken advantage of, it is important to understand what is the time scale at which takes place. Also the detailed mechanisms by which the market takes into account the expected future events will be studied in detail.

The detailed comparison between the models of behavior of the individual traders in stock markets and the actual transactions data requires the design of measurements which are more discriminating than the usual probability distribution functions and their momenta.

The new generation of measurement will address more directly the actual behavior of the market participants.

For instance one can compare measurements in the model vs. real data in:

- the size distribution the individual steps sizes (for the entire population and for individual investors),
- the probability distribution of short-selling vs. long positions,
- the distribution of and the individual trades volume,
- the time persistence of periods with large/small such volumes,
- the distribution of the time spans during which the traders keep a position open and for what volume,
- the trade frequency of various traders – the distribution of these frequencies in the population,
- the time persistence of trade frequencies over time periods,

- the relation between trade frequency and trade volumes,
- the evidence of division of large operations in smaller separate deals,
- the time persistence of up-down trends,
- the influence of an individual trade on subsequential trades price trends as a function of the individual trade volume.

Such measurements will give a fuller picture of the mechanisms by which the collective market trends emerge from the individual (“microscopic”) trading behavior.

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